Global Initiative of Academic Networks (GIAN)

Presents a Course on

# "Generalized Iterative Methods for Nonlinear Differential Equations"

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## Overview

Mathematical modeling of real life problems in science, engineering and other branches including social science leads to nonlinear dynamic systems with initial and /or boundary conditions. Quantitative and qualitative properties of these dynamic systems, such as existence, uniqueness play an important role in applying these models to our advantage. The explicit computation of solutions for the nonlinear problem is rarely possible. However, the method of lower and upper solutions is an effective tool to prove the existence results for nonlinear dynamic systems. These lower and upper solutions not only act as bounds for the solution but also assure the interval of existence for initial value problems. Monotone sequences that converge uniformly and monotonically to the minimal and maximal solutions can be generated by effectively associating the method of lower and upper solutions with monotone method. This method scales down the original problem to a computationally easier linear problems. If further uniqueness conditions are satisfied, the minimal and maximal solutions converge to the unique solution of the nonlinear problem. Through this iterative method, the existence of the solution can be confirmed both theoretically and computationally. However, the monotone method is useful when the forcing term is either increasing or can be made increasing by an addition of a linear term. In order to overcome this problem, coupled lower and upper solutions will be introduced. In this case, the monotone method is referred to as generalized monotone method. The generalized monotone method yields monotone sequences which converge to coupled minimal and maximal solutions. The rate of convergence of the iterates for monotone method or generalized monotone method is linear, whereas, the method of quasilinearization or generalized quasilinearization yields quadratic convergence. In this case, we assume a stronger condition on the nonlinear term, namely convexity, concavity or both. As a byproduct, the iterates converge to the unique solution quadratically.

This compact course focuses on introducing the participants to the method of lower and upper solutions (coupled lower and upper solutions) combined with an iterative method for obtaining existence results for a wide range of nonlinear problems. The lectures are outlined to start from the very basic concepts and move towards providing proper training to apply the theory to various real life problems. This course will act as a platform to learn the foundations of dealing with nonlinear problems with applications. In addition, it promotes future collaborative activities between researchers from varied disciplines. Participants will also gain much by carefully applying this seemingly different topic in their own research fields. The technical sessions are given prominence to make sure that it will be fruitful for all the participants.

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| **Modules** | Schedule Dates: December 18-22, 2017<br>Lectures: 10:00 am to 1:00 pm; Tutorials: 3:00 pm to 5:00 pm<br>**Number of participants for the course will be limited to fifty.** |
| **You Should Attend If…** | Students at all levels (BTech/MSc/MTech/PhD), engineers, researchers and faculty from academic and technical institutions. |
| **Fees** | **Bachelor's or Master's students: Rs. 1000; Research Scholars: Rs. 2000**<br>**Faculty, Scientists, Engineers and delegates from Industry: Rs. 3000**<br>**Participants from abroad: $200**<br>The above fee includes lunch, instructional materials, 24 hours internet facility. The participants will be provided with accommodation on payment basis. |

# The Faculty

**Professor Aghalaya S Vatsala** is a Professor at the University of Louisiana at Lafayette since 1996. After her Master's degree from Bangalore University in 1968, she completed her doctoral degree from IIT Madras in 1973. She worked as C.S.I.R postdoctoral fellow for two years at IIT, Madras. She served as a faculty at the Regional College of Education, (NCERT) Mysore. She moved to United States in 1978, and worked as a visiting faculty at University of Texas at Arlington, Texas, and Oklahoma State University, Stillwater, Oklahoma. She joined University of Louisiana at Lafayette in 1984, and currently serving as Pennzoil endowed Professor of Mathematics, in the Mathematics department. Professor Aghalaya S. Vatsala is one of the pioneers in developing monotone iterative methods for nonlinear differential equations. Her contributions in the area of ordinary and partial differential equations made her well known in the mathematics world. She has successfully completed various sponsored research projects including two sponsored research projects from U.S. Army. She has successfully organized more than twenty five workshops/conferences including special sessions in second, third and fourth World Congress of Nonlinear Analysts and special session in various American Mathematical Society Annual Conferences.

Aghalaya S. Vatsala has also served as an editorial member for various international journals including Nonlinear Analysis: Hybrid Systems (2009-2011) and Nonlinear Analysis: Theory, Methods & Application Series A & C (till 2011). She is also a member of various international academic bodies including Board of Experts for International Federation of Nonlinear Analysis. She has published more than 125 research papers in reputed international journals. She is also a reviewer for various mathematical databases and a co-author of various research monographs. One of the research monographs entitled 'Monotone Iterative Technique for Nonlinear Differential Equations' co-authored with G.S. Ladde and V. Lakshmikantham is cited more than thousand times as per google scholar statistics. She has supervised twelve students for their PhD degree. She has been honored with UL Lafayette foundation distinguished professor award in 2014. Apart from delivering numerous presentations in conferences, she also has motivated students by her talks about career in mathematical sciences as a part of outreach program.

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**Dr. V. Antony Vijesh** received his PhD degree from IIT Madras in 2007 and is currently working as an Associate Professor at IIT Indore. His broad area of research is nonlinear analysis and numerical analysis. Presently his group is developing efficient numerical methods for partial differential equations and integro-differential equations using finite difference method and wavelets. The other work aspect of this group is to study the existence and uniqueness of nonlinear differential equations via various iterative schemes and analyze their convergence behavior with systematic error estimates. Their findings have been published in reputed international journals.

## Duration:

**18-22 December, 2017**

## Course Coordinator:

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| Schedule | Lectures (Morning Session) | Tutorials (Afternoon Session) |
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| 18 December 2017 | **Lecture 1:** Introduction to basic results and definitions<br>**Lecture 2:** Monotone method and generalized monotone method for system of first order ordinary differential equations with initial conditions | Based on lectures, differential equation arising from population model will be discussed in detail. |
| 19 December 2017 | **Lecture 3:** Comparison results for scalar parabolic differential equations and its applications<br>**Lecture 4:** Existence of coupled minimal and maximal solutions for nonlinear parabolic differential equations via generalized monotone method | Based on lectures, differential equation from combustion theory and population model will be discussed in detail. Numerical methods based on Lecture 4 will also be discussed. |
| 20 December 2017 | **Lecture 5:** Comparison results for system of parabolic differential equations and its applications<br>**Lecture 6:** Existence of coupled minimal and maximal solutions for system of nonlinear parabolic differential equations via generalized monotone method | Based on lectures, differential equation arising from Belousov-Zhabotinskii reaction and Predictor-Prey model will be discussed in detail. Numerical methods based on Lecture 6 will also be discussed. |
| 21 December 2017 | **Lecture 7:** Comparison principle for hyperbolic differential equations and its applications; existence and uniqueness results related to linear hyperbolic partial differential partial differential equations with initial and boundary conditions.<br>**Lecture 8:** Existence and uniqueness results for system of nonlinear hyperbolic differential equations via monotone iterations. | Problem solving session with examples based on Lecture 7 and Lecture 8, nonlinear Klein-Gordon equation and Darboux problem will be discussed in detail. Numerical methods based on Lecture 8 |
| 22 December 2017 | Group discussions on recent trends in monotone iterative technique. | |